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Computer Calculation and Analysis of the P4SR Heat-Stress Index

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A method for the accurate calculation of the P4SR Index of heat stress by computer is described. Analysis shows that, although the P4SR Index is basically sound, at least one serious imperfection exists, namely, the effect attributed to an air speed of 75 ft/min. The only previously described arithmetical method for the calculation of the index is examined and found to be unacceptable.

The Predicted Four-hour Sweat-rate (P4SR) Index of heat stress (McArdle, Dunham, Holling, Ladell, Scott, Thompson, and Weiner, 1947) has been widely used since its introduction more than 20 years ago, and has been shown to be one of the most useful of the many indices of heat stress (Macpherson, 1962; Mehta and Niyogi, 1970). Its special advantage is that, in estimating the heat stress imposed by a given environment, it takes into consideration the metabolic rate and the clothing of those exposed to it. This is of particular importance in industry where both the clothing and the metabolic rate may differ widely from one task to another.

Its chief disadvantage is that its calculation requires the use of a complex nomogram. This is extremely time consuming when large numbers of calculations have to be made.

This laboratory recently undertook a large-scale field survey of heat stress in New Guinea, and the analysis of the results required the determination of the P4SR Index for several thousand sets of observations. To lessen the labour involved in this project, a computational method was devised, which is no less accurate, but infinitely less laborious than the original method.

The computational method is described in this paper, and the only previously described arithmetical method (Wyndham, Allan, Bredell, and Andrew, 1967) is discussed.

ALGORITHM

The calculation of the P4SR Index as originally described proceeds in three stages. First, a modified wet-bulb temperature (MWB) is calculated by adding three quantities to the observed wet-bulb temperature (WB). The first increment depends on the difference between the globe-thermometer (GT) and dry-bulb (DB) temperatures. It is given by the expression 0.4 (GT − DB). The second depends on clothing, and is 1.8°F when the clothing consists of cotton drill coveralls weighing 1.0 kg. The third increment depends on the metabolic rate (M), and is obtained from a small inset chart on the standard nomogram.

The second stage is the determination of the Basic Four-hour Sweat-rate (B4SR). This is done by entering a nomogram with three arguments: the observed air-speed (AS), the observed globe-thermometer temperature, and the calculated MWB.

The third stage is the determination of the P4SR Index itself. This is done by adding to the B4SR amounts which depend on the metabolic rate (M) and the clothing (C).

Development of the Algorithm

For any specified AS and MWB, values may be obtained from the B4SR nomogram which yield a curve relating GT to B4SR for the specified AS and MWB. Thus, for each MWB, a family of curves may be obtained which shows the effect of air speed on the GT–B4SR relationship. Two typical families of curves are reproduced. Figure 1 shows the curves for MWB = 90°F and Fig. 2 shows those for MWB = 95°F. It will be seen that the disposition of the component curves of the two families is quite different although they are separated by only 5°F MWB. It will be apparent that for any set of conditions, the MWB determines which family of curves is to be chosen, and the AS selects a particular curve from the family.

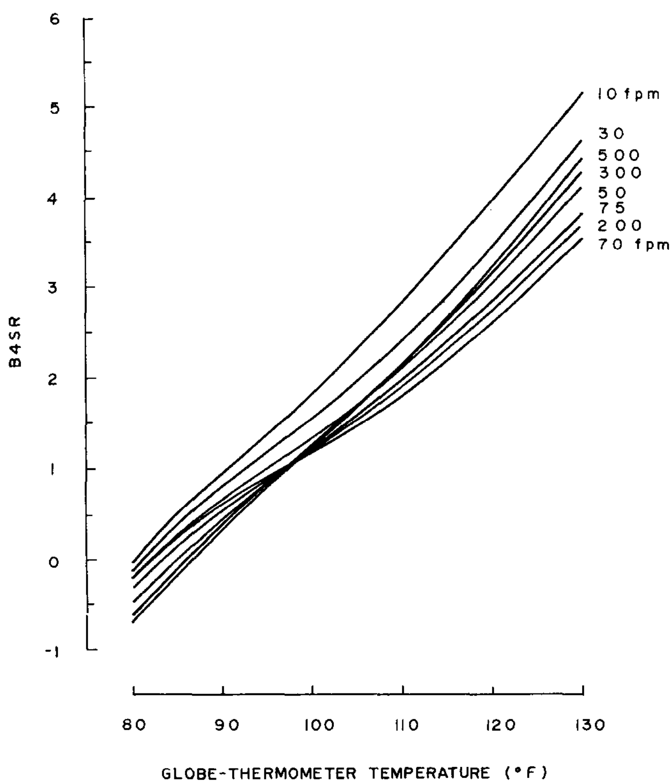

FIG. 1. Relation between B4SR and globe-thermometer temperature for modified wet-bulb temperature of 90°F.

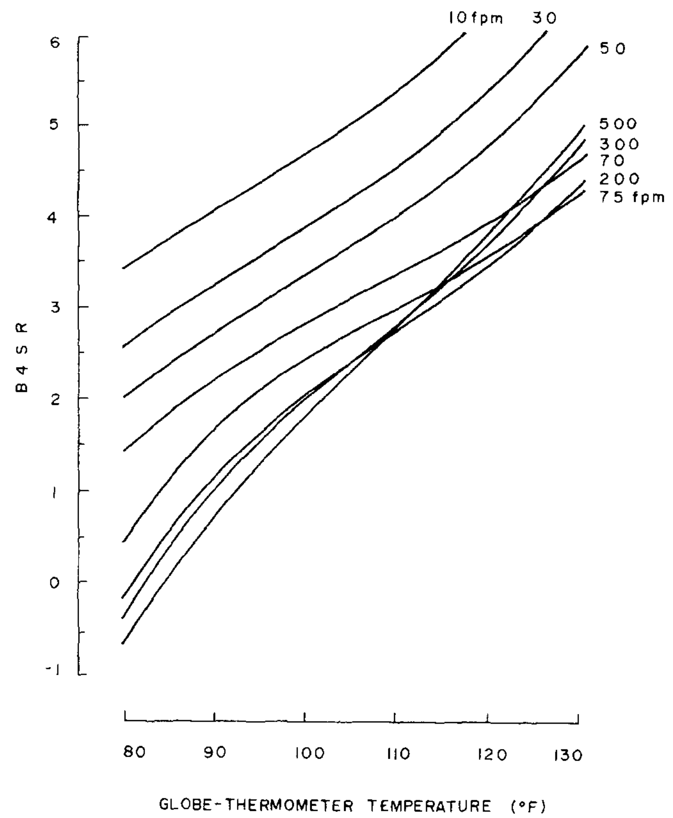

FIG. 2. Relation between B4SR and globe-thermometer temperature for modified wet-bulb temperature of 95°F.

The P4SR is determined from the B4SR, as noted above, by the addition to the B4SR of an amount determined by the metabolic rate and the clothing. M and C, therefore, have two effects. The first is they modify the wet-bulb temperature and thereby determine to which family of curves the B4SR belongs. The second is they determine the P4SR, which they do by shifting the selected family of B4SR curves upward without changing the shape of the component curves.

Inspection of the curves for the B4SR suggested that they could be well described by third-order polynomial equations, and this proved to be so. To obtain these equations, for the eight air speeds (10, 30, 50, 70, 75, 200, 300, and 500 ft/min) for which there are B4SR curves on the nomogram, values for GT were read from the nomogram for each graduation marked on the B4SR curve and MWB scale corresponding to the air speed under consideration. Since the GT scale is linear, it is more accurate to interpolate on this scale for fixed values of B4SR and MWB rather than interpolate on the B4SR curves (which are nonlinear), as is required in normal use of the nomogram.

For each air speed, the values for GT and B4SR thus obtained for 15 selected MWB temperatures (60, 80, 84, 87, and 89 through 99°F) were then fitted by

TABLE I
Comparison Between Computed B4SR Values and Those Obtained from the Nomogram for AS = 500 ft/min at MWB = 95°F

GT	B4SR Observed	B4SR Computed	Error
82.8	−0.25	−0.289	−0.039
84.7	0.0	0.009	0.009
88.2	0.5	0.517	0.017
92.3	1.0	1.046	0.046
96.7	1.5	1.553	0.053
100.4	2.0	1.940	−0.060
105.6	2.5	2.444	−0.056
111.5	3.0	2.990	−0.010
117.1	3.5	3.517	0.017
122.0	4.0	4.011	0.011
127.3	4.5	4.607	0.107
129.7	5.0	4.906	−0.094

the method of least squares. Thus, 15 equations were obtained for each air speed, and since each equation requires four coefficients, there are 480 coefficients in all. The 15 MWB values were selected to give a reasonably linear distribution of temperatures, particularly at the lower ends of the MWB scales.

As a check on accuracy, the entire array of B4SR values was computed using the complete matrix of 8×15 equations. The greatest error was found to be 0.107, which occurred in the 500 ft/min equation in the 95°F MWB family, at 127.3° GT. Table I lists the observed and calculated values for this equation. In no other case did the error exceed 0.1, and only occasionally was it in excess of 0.05.

In Fig. 3, superimposed on the hand-drawn curves of Fig. 2 (the 95°F MWB family), is a set of curves plotted by computer to the same scale as the original drawing. It will be seen that the correspondence is satisfactory.

The Computer Programme

A function subroutine has been written in Fortran 63 to perform the sequence of calculations. The routine returns the P4SR to the calling programme for arguments DB, WB, GT, AS, and C. When used with a CDC 3200 machine not equipped with floating-point hardware (add time 2.5 μsec), execution time is 30–76 msec and, as compiled by the standard Control Data Fortran compiler, the routine requires 778 48-bit memory locations, of which 480 are occupied by the coefficients.

The P4SR Index is computed by the routine in the same three stages as when the Index is obtained from the nomogram. In the first stage the modified wet-bulb temperature is calculated. This is obtained by adding three corrections to WB. The first is due to GT, and is 0.4 (GT − DB) as noted above. The second is due to clothing, and the routine assumes that this correction is linearly dependent on the weight of the clothing. The correction is then 1.8 C°F, where C is the weight of the clothing in kilograms. The third correction is due to M. It can be

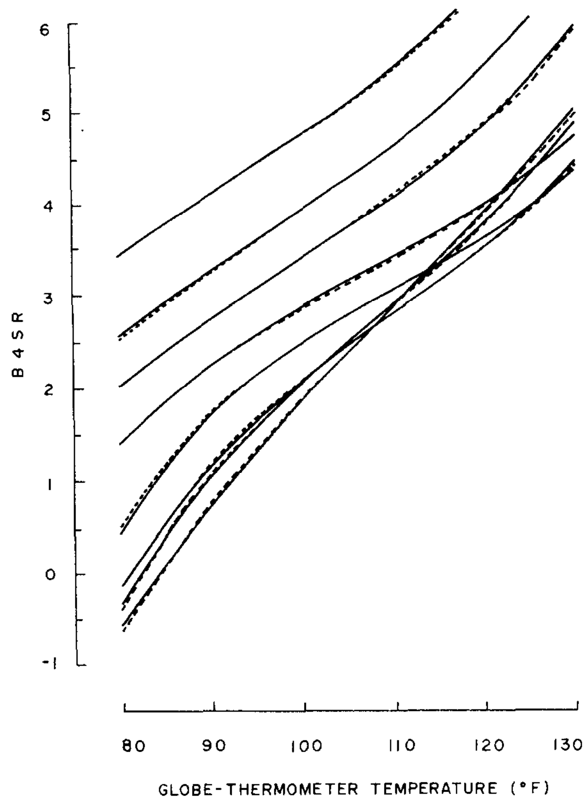

FIG. 3. Family of hand-drawn curves for modified wet-bulb temperature of 95°F (solid lines) with computer-drawn curves (broken lines) superimposed.

shown by reference to the inset chart on the B4SR nomogram that if M exceeds 100 kcal m^{-2} hr^{-1}, then this correction is (0.03 M + 1)°F; if M is less than 100 kcal m^{-2} hr^{-1} it is (0.087 M — 4.7)°F, and if M is exactly 100, then the correction is 4.0°F. In the computation, the value of M is tested, the appropriate correction calculated, and added to WB together with those due to GT and C.

In the second stage the B4SR is calculated. The routine selects the coefficients appropriate to AS and MWB. These are applied to GT according to the equation

$$B4SR = \sum_i c_i GT^{i-1} (i = 1,2,3,4).$$

where the c_i are the coefficients from the appropriate polynomial equation.

In the third stage of the computation, the increments due to M and C are calculated and added to the B4SR to give the P4SR. The increment due to M is 0.014 (M — 54), where M is the metabolic rate in kcal m^{-2} hr^{-1}. The increment due to clothing is 0.25 + 0.006 (M — 54) when the clothing consists of cotton drill coveralls weighing 1.0 kg. The routine assumes again that the effect is linearly related to the weight of the clothing, so that the combined increment due to M and C becomes 0.25 C + (0.014 + 0.006 C)(M — 54), where C is the weight of clothing in kilograms.

In most cases the air speed and/or MWB will not coincide with values for which coefficients are available. The routine computes the index for tabulated values of AS and MWB immediately above and below the required values, and interpolates linearly within the area so defined. For example, if AS and MWB were 100 ft/min and 94.5°F, respectively, the index would be calculated for the four points corresponding to air speeds of 75 and 200 ft/min, with MWB of 94 and 95°F. Linear interpolation within these four points then yields the P4SR Index for the required values.

EXAMINATION OF THE P4SR INDEX

The computer can also conveniently be used to investigate the P4SR Index. By varying one parameter and holding all others constant, it can be made to plot the calculated P4SR Index against the varying parameter. In Fig. 4, three such plots of P4SR against AS are shown for different combinations of WB and DB. In each of these plots, GT is assumed to be the same as DB; C is zero, and M is 54 kcal m^{-2} hr^{-1}. Under these conditions the P4SR is equal to the B4SR.

The upper curve shows that for hot humid conditions, increasing the air speed produces rapid reduction of heat stress until an air speed of 200 ft/min is at-

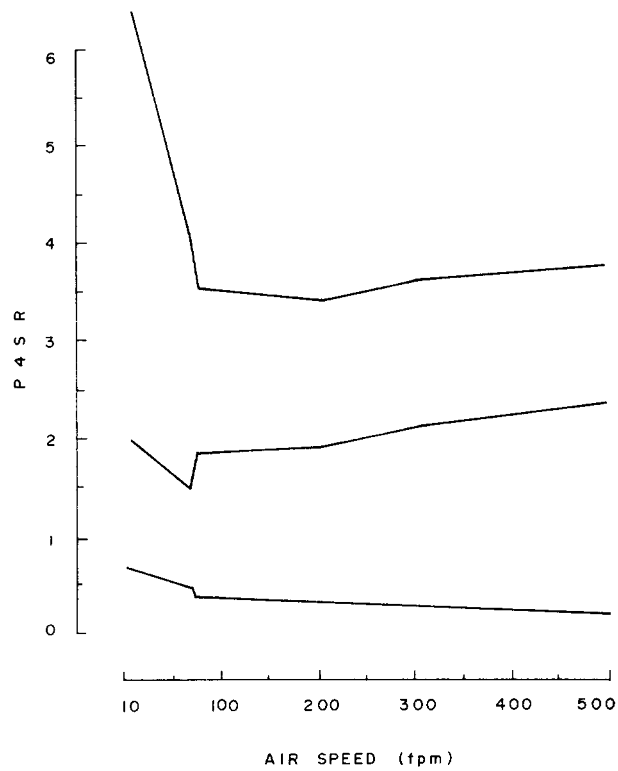

FIG. 4. Computer-drawn curves showing relation between P4SR and air speed for three combinations of wet- and dry-bulb temperatures. *Upper curve*: WB = 95°F, DB = 120°F; *middle curve*: WB = 75°F, DB = 120°F; *lower curve*: WB = 88°F, DB = 90°F. In all curves, M = 54 kcal m^{-2} hr^{-1}, C = 0, and GT = DB.

tained, after which there is a gradual increase in the heat stress. For hot dry conditions (middle curve), there is less rapid initial amelioration, and the turning point is less than 100 ft/min. For relatively cool humid conditions (lower curve), there is a gradual but progressive amelioration of heat stress up to 500 ft/min. These results would be expected from examination of published data (Macpherson, 1960), and suggest that for these conditions at least, the P4SR Index reliably indicates the heat stress.

The curves of Fig. 4 reveal an anomaly in the nomogram. It seems unlikely that the sharp bend in the upper and lower curves at 75 ft/min is a true representation. It would be more reasonable to expect a smooth curve. The middle curve of Fig. 4 implies that the turning point is 70 ft/min, and that there is a rapid worsening of the heat stress until 75 ft/min is reached, after which there is an abrupt change in the effect of increasing air speed. It seems most unlikely that a change in air speed of 5 ft/min (from 70 to 75 ft/min) could produce this effect. Many similar plots have been examined, and all show a sharp bend at 75 ft/min.

It is suggested that the nomogram values for 75 ft/min are in error, and that by omitting the equations for this air speed, and by using nonlinear interpolation, a method of calculation producing more realistic values would be obtained. But without extensive experimental investigation, such a modification is not entirely justified, as it is not inconceivable that the equations for 75 ft/min are correct, while those for adjacent air speeds are in error.

Examination of Another Method of Calculation

The only previously published method for the arithmetical calculation of the P4SR Index (Wyndham *et al.*, 1967) was investigated. This method utilises a single equation which was derived by multiple regression analysis. The authors state that the correlation coefficient between values obtained from the nomogram and those calculated from the equation is 0.9795, but they do not indicate the standard error of the estimate. This has been found to exceed 0.5 when analysis is confined to the 243 sets of values from which the equation was derived, so that within this range, estimates yielded by the equation are unacceptable. It should be remembered that a high value for the correlation coefficient merely indicates that two vectors under consideration point in the same direction; it sheds little light on the numerical correspondence between the ordered components of the vectors. The statement that the equation accounts for 96% of the variation is mathematically naive.

Outside the range of environmental conditions for which the equation was derived, results obtained with it were often ludicrous. For example, at DB = 80°F, WB = 60°F, GT = 84°F, and AS = 50 ft/min, the calculated P4SR Index for a metabolic rate of 54 kcal m^{-2} hr^{-1} was —5.84, and increasing M to 175 kcal m^{-2} hr^{-1} reduced the calculated value to —8.13. The environmental conditions of this example are in no way extreme, and the results obtained clearly show that the method is useless for general use.

Reasons for the failure of the equation are not hard to find. An obvious one is that it is a second-order polynomial equation, and it will be clear that the curves of Figs. 1 and 2 cannot be described by an equation of this order.

Using this highly unsatisfactory equation, Wyndham *et al.*, investigated the effect of GT and WB by obtaining partial derivatives with respect to these variables. They were led to the belief that the relative effect of GT and WB depends on humidity, when in fact it is constant (0.4). Because the equation led to this erroneous deduction, they came to the curious conclusion that the P4SR Index itself was faulty.

POSSIBLE REFINEMENT OF THE METHOD

The method described in this paper will give values for the P4SR Index over the full range covered by the nomogram with error usually less than 0.05 when compared with values obtained directly from the nomogram.

During development of the method it was expected that it would be possible to combine the curves of any family into a two-dimensional surface curved in three dimensions, and then to combine the several families into a three-dimensional "surface" curved in four dimensions, so that the Index could be calculated by multidimensional methods using perhaps no more than 20 coefficients. Unfortunately this proved to be impracticable. An isometric representation of a typical family of curves (that for MWB = 95°F) is reproduced in Fig. 5. The sharp fold in this surface at AS = 75 ft/min shows where the difficulty was encountered. This fold corresponds to the anomalies noted above, and lends weight to the belief that the nomogram is faulty in this area.

If experimental investigation can eliminate this fault and permit the families of curves to be represented as smoothly curved surfaces, then it will be possible to devise a more elegant method of computation than that described in this paper.

For this and other reasons, including that of space, the computer programme is not reproduced here but photocopies of the listing are available from the author on request.

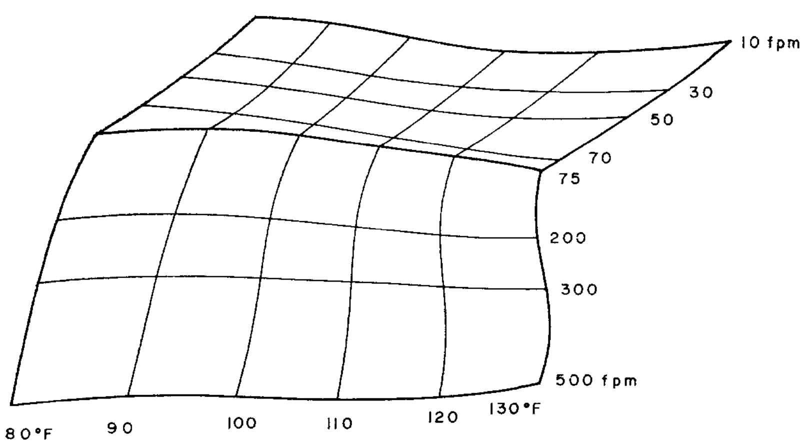

FIG. 5. Isometric representation of the family of curves for MWB = 95°F.

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